Thus, for the tube flow depicted in Figure 5.9 the total energy added can be expressed in terms of a bulk-temperature difference by

$$q = \dot{m}c_p(T_{b2} - T_{b1}) = hA(T_w - T_b)_{av} \qquad 5.67$$

$$T_b = \frac{T_{b1}+T_{b2}}{2}$$

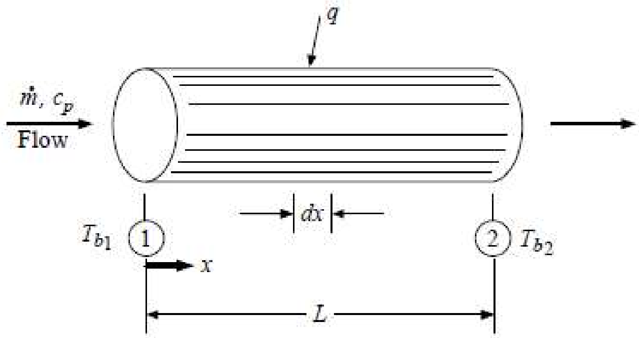


Figure 5.9 Total heat transfer in terms of bulk-temperature difference.

- ## Empirical Relation In Laminar Flow In Tube

Constant Heat Flux

Fully developed laminar flow in circular tube

$$Nu_D = \frac{hD}{k} = 4.36 \qquad 5.68$$

Constant Surface Temperature

Fully developed laminar flow in circular tube

$$Nu_D = \frac{hD}{k} = 3.66 \qquad 5.69$$

Laminar flow in the Entrance Region

$$\overline{Nu}_D = \frac{hD}{k} = 3.66 + \frac{0.065\left(\frac{D}{L}\right)Re\,Pr}{1+0.04\left[\left(\frac{D}{L}\right)Re\,Pr\right]^{\frac{2}{3}}} \qquad 5.70$$

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature. The average Nusselt number for developing laminar flow in a circular tube in that case can be determined from [Sieder and Tate (1936),

$$\overline{Nu}_D = 1.86\left(\frac{Re_D \Pr D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad \frac{Re_D \Pr D}{L} > 10 \qquad 5.71$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_w, which is evaluated at theT_w.

Entry Length

$L_{t,laminar} \approx 0.05 Re_D Pr D$

EXAMPLE 8–3 Flow of Oil in a Pipeline through a Lake page 439 in the book.

- **Imperical Relations for Turbulent Flow in Tube**

For fully developed, smooth surface and moderate temperature differences:

$Nu_D = 0.023 Re_D^{0.8} Pr^n \qquad 0.6 < Pr < 100 \qquad 5.72$

The properties in this equation are evaluated at the average fluid bulk temperature, and the exponent n has the following values:

$$n = \begin{cases} 0.4 & \text{for heating of the fluid} \\ 0.3 & \text{for cooling of the fluid} \end{cases}$$

More recent information by Gnielinski suggests that better results for turbulent flow in smooth tubes may be obtained from the following:

$$Nu_D = 0.0214(Re_D^{0.8} - 100)Pr^{0.4} \qquad \begin{cases} 0.5 < Pr < 1.5 \\ 10^4 < Re_D < 5 * 10^6 \end{cases} \qquad 5.73$$

Or

$$Nu_D = 0.012(Re_D^{0.87} - 280)Pr^{0.4} \qquad \begin{cases} 1.5 < Pr < 500 \\ 10^3 < Re_D < 10^6 \end{cases} \qquad 5.74$$

To take into account the property variations, Sieder and Tate recommend the following relation:

$$Nu_D = 0.027 Re_D^{0.8} Pr^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \qquad 5.75$$

For entrance region

The above equations are apply to fully developed turbulent flow in tubes. In the entrance region the flow is not developed, and Nusselt recommended the following equation:

$$Nu_D = 0.036 Re_D^{0.8} Pr^{\frac{1}{3}} \left(\frac{D}{L}\right)^{0.055} \qquad \text{For } 10 < \frac{L}{D} < 400 \qquad 5.76$$

Entry Length

$L_{t,turbulent} \approx 10D$

The above equations offer simplicity in computation, but uncertainties on the order of ±25 percent are not uncommon. Petukhov has developed a more accurate, although more complicated, expression for fully developed turbulent flow in smooth tubes:

$$Nu_D = \frac{\left(\frac{f}{8}\right) Re_D Pr}{1.07 + 12.7\left(\frac{f}{8}\right)^{0.5}\left(Pr^{\frac{2}{3}} - 1\right)}\left(\frac{\mu_b}{\mu_w}\right)^n \qquad 5.77$$

Where:

$n = 0.11$ for $T_w > T_b$,

$n = 0.25$ for $T_w < T_b$, and

$n = 0$ for constant heat flux or for gases.

All properties are evaluated at

$T_f = \frac{(Tw + Tb)}{2}$ except for μ_b and μ_w.

The friction factor may be obtained either from the following for smooth tubes:

$$f = (1.82 \log_{10} Re_D - 1.64)^{-2} \qquad 5.78$$

Equation (5.77) is applicable for the following ranges:

$0.5 < Pr < 200$ for 6 percent accuracy

$0.5 < Pr < 2000$ for 10 percent accuracy

$10^4 > Re_D > 5 * 10^6$

$0.8 < \mu_b/\mu_w < 40$

All above equations are for smooth pipes. For rough pipe we use the equation below:

$$St_b Pr_f^{\frac{2}{3}} = \frac{f}{8}$$

$$\frac{Nu_D}{Re_D Pr} Pr_f^{\frac{2}{3}} = \frac{f}{8}$$

The friction coefficient *f* is defined by

$$\Delta p = f \frac{L}{D} \rho \frac{u_m^2}{2}$$

An empirical relation for the friction factor for rough tubes is given as

$$f = \frac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D}\right) + \frac{5.74}{Re_D^{0.9}}\right]^2}$$

For $10^{-6} < \frac{\varepsilon}{D} < 10^{-3}$, and $5000 < Re_D < 10^8$

- Isothermal Parallel Plates

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length L is expressed as (Edwards et al., 1979)

$$Nu_{D_h} = \frac{hD_h}{k} = 7.45 + \frac{0.03\left(\frac{D_h}{L}\right)Re_{D_h}\,Pr}{1+0.016\left[\left(\frac{D_h}{L}\right)Re_{D_h}\,Pr\right]^{\frac{2}{3}}} \qquad \text{entry region, laminar flow} \quad Re \leq 2800$$

$D_h = 2S$

Where

S : is the space between the two plates.

Example 5.9: Turbulent Heat Transfer in a Tube

Air at 2 atm and 200◦C is heated as it flows through a tube with a diameter of 1 in (2.54 cm) at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20◦C above the air temperature, all along the length of the tube. How much would the bulk temperature increase over a 3-m length of the tube?

Solution

We first calculate the Reynolds number to determine if the flow is laminar or turbulent, and then select the appropriate empirical correlation to calculate the heat transfer. The properties of air at a bulk temperature of 200◦C are

$$\rho = \frac{p}{RT} = \frac{(2)(1.0132*10^5)}{(287)(473)} = 1.493 \text{ kg/m}^3$$

$Pr = 0.681$, $\mu = 2.57 * 10^{-5}$ kg/m.s, $k = 0.0386$ W/m.˚C, $c_p = 1.025$ kj/kg. ˚C.

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{(1.493)(10)(0.0254)}{2.57*10^{-5}} = 14756$$

So that the flow is turbulent.

Check the entry length

$L_{t,turbulent} \approx 10D = 10(0.0254) = 0.254$ m

It's too shorter than the length of the tube. Therefore the flow assumed fully developed. We therefore use Equation (5.72) to calculate the heat-transfer coefficient.

$$Nu_D = 0.023Re_D^{0.8}Pr^n$$

For heating the fluid n=0.4

$Nu_D = 0.023(14756)^{0.8}(0.681)^{0.4} = 42.67$

$h = \frac{Nu_D k}{D} = \frac{(42.67)(0.0386)}{0.0254} = 64.85$ W/m².°C.

$\frac{q}{L} = h\pi D(T_w - T_{b)} = (64.85)(\pi)(0.0254)(20) = 103.5$ W/m.

$q = \dot{m}c_p\Delta T_b = L\left(\frac{q}{L}\right)$

$\dot{m} = \rho u_m \frac{\pi D^2}{4} = (1.493)(10)(\pi)\frac{(0.0254)^2}{4} = 7.565 * 10^{-3}$ kg/s.

$\Delta T_b = \frac{L\left(\frac{q}{L}\right)}{\dot{m}c_p} = \frac{(3)(103.5)}{(7.565*10^{-3})(1025)} = 40.04$ °C.

EXAMPLE 5.10: Heating of Water in Laminar Tube Flow

Water at 60◦C enters a tube of 1-in (2.54-cm) diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80◦C.

Solution

We first evaluate the Reynolds number at the inlet bulk temperature to determine the flow regime.

The properties of water at 60◦C are

$\rho = 985$ kg/m³

$Pr = 3.02$, $\mu = 4.71 * 10^{-4}$ kg/m.s, $k = 0.651$ W/m.°C, $c_p = 4.18$ kj/kg. °C.

$Re_D = \frac{\rho u_m D}{\mu} = \frac{(985)(0.02)(0.0254)}{4.71*10^{-4}} = 1062$

So the flow is laminar.

Check for entry length

$L_{t,laminar} \approx 0.05 Re_D Pr D = (0.05)(1062)(3.02)(0.0254) = 4$ m

Therefore the flow not fully developed (i.e the flow in the interence region)

Calculating the additional parameter, we have

$\frac{Re_D \Pr D}{L} = \frac{(1062)(3.02)(0.0254)}{3} = 27.15 > 10$

So Equation (5.71) or Equation (5.70) is applicable. We do not yet know the mean bulk temperature to evaluate properties so we first make the calculation on the basis of 60◦C,

At the wall temperature of 80◦C we have

$\mu_w = 3.55 * 10^{-4}$ kg/m.s.

$$\overline{Nu}_D = 1.86\left(\frac{Re_D \Pr D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\overline{Nu}_D = 1.86\left(\frac{(1062)(3.02)(0.0254)}{3}\right)^{\frac{1}{3}}\left(\frac{4.41}{3.55}\right)^{0.14} = 5.816$$

$$h = \frac{Nu_D k}{D} = \frac{(5.816)(0.651)}{0.0254} = 149.1 \text{ W/m}^2.^\circ\text{C.}$$

$$\dot{m} = \rho u_m \frac{\pi D^2}{4} = (985)(0.02)(\pi)\frac{(0.0254)^2}{4} = 9.982 * 10^{-3} \text{ kg/s.}$$

$$q = h\pi DL\left(T_w - \frac{T_{b1}+T_{b2}}{2}\right) = \dot{m}c_p(T_{b2} - T_{b1})$$

$$(149.1)(\pi)(0.0254)(3)\left(80 - \frac{60+T_{b2}}{2}\right) = (9.982 * 10^{-3})(4180)(T_{b2} - 60)$$

$$T_{b2} = 71.98\ ^\circ\text{C}$$

$$T_{b,avg} = \frac{T_{b1}+T_{b2}}{2} = \frac{60+71.98}{2} = 66\ ^\circ\text{C}$$

$$\rho = 982 \text{ kg/m}^3$$

$Pr = 2.78$, $\mu = 4.36 * 10^{-4}$ kg/m.s, $k = 0.656$ W/m.°C, $c_p = 4.185$ kj/kg. °C.

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{(982)(0.02)(0.0254)}{4.63*10^{-4}} = 1147$$

$$\frac{Re_D \Pr D}{L} = \frac{(1147)(2.78)(0.0254)}{3} = 27.00 > 10$$

$$\overline{Nu}_D = 1.86\left(\frac{Re_D \Pr D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\overline{Nu}_D = 1.86(27)^{\frac{1}{3}}\left(\frac{4.36}{3.55}\right)^{0.14} = 5.743$$

$$h = \frac{Nu_D k}{D} = \frac{(5.743)(0.656)}{0.0254} = 148.3 \text{ W/m}^2.^\circ\text{C.}$$

$$(148.3)(\pi)(0.0254)(3)\left(80 - \frac{60+T_{b2}}{2}\right) = (9.982 * 10^{-3})(4185)(T_{b2} - 60)$$

$$T_{b2} = 71.88\ ^\circ\text{C}$$

5.9 Flow Across Cylinders And Spheres

As the flow progresses along the front side of the cylinder, the pressure would decrease and then increase along the back side of the cylinder, resulting in an increase in free-stream velocity on the front side of the cylinder and a decrease on the back side. The transverse velocity (that velocity parallel to the surface) would decrease from a value of u_∞ at the outer edge of the boundary layer to zero at the surface. As the flow proceeds to the back side of the cylinder, the pressure increase causes a reduction in velocity in the free stream and throughout the boundary layer.

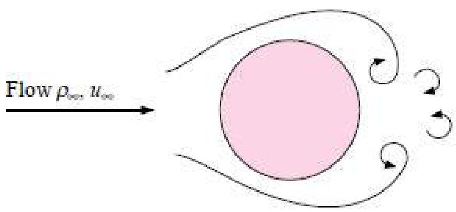


Figure 5.10 Cylinder in cross flow.

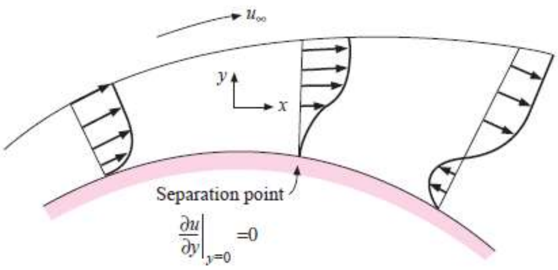


Figure 5.11 Velocity distributions indicating flow separation on a cylinder in cross flow.

The pressure increase and reduction in velocity are related through the Bernoulli equation written along a streamline:

$$\frac{dp}{\rho} = -d\left(\frac{u^2}{2g_c}\right)$$

When the velocity gradient at the surface becomes zero, the flow is said to have reached a separation point:

seperation point at $\left.\frac{\partial u}{\partial y}\right]_{y=0} = 0$

The drag coefficient for bluff bodies is defined by

$$\text{Drage force} = F_D = C_D A \frac{\rho u_\infty^2}{2g_c} \qquad 5.79$$

where C_D is the drag coefficient and

A is the *frontal area* of the body exposed to the flow, which, for a cylinder, is the product of diameter and length.

$A = LD$

The values of the drag coefficient for cylinders and spheres are given as a function of the Reynolds number in Figures 5.12 and 5.13.

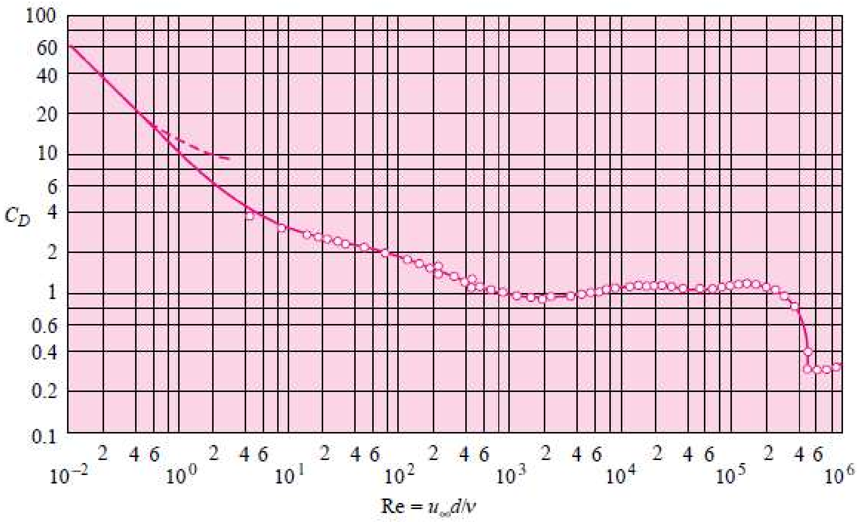


Figure 5.12: Drag coefficient for circular cylinders as a function of the Reynolds number.

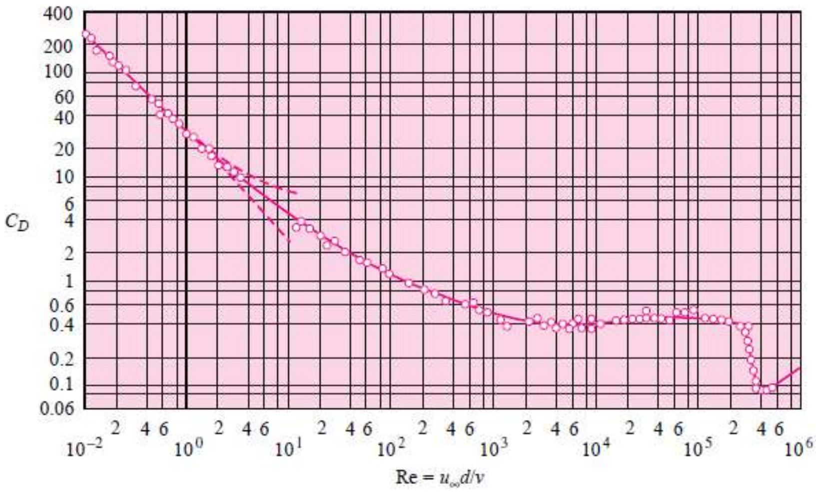


Figure 5.13: Drag coefficient for spheres as a function of the Reynolds number.

5.9.1 Cylinder

The resulting correlation for average heat-transfer coefficients in cross flow over circular cylinders is

$$Nu_{Df} = \frac{hD}{k_f} = C\left(\frac{u_\infty D}{v_f}\right)^n Pr_f^{\frac{1}{3}} \qquad \text{(flow of gas)} \qquad 5.80$$

where the constants C and n are tabulated in Table 5.1. Properties for use with Equation (5.80) are evaluated at the film temperature as indicated by the subscript f .

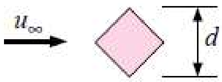
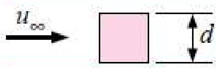
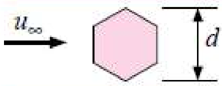
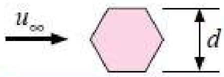
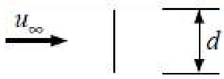
Table 5.1 Constants for use with Equation (5.80),

Re_{Df}	C	n
0.4–4	0.989	0.330

4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

For non-circular cylinder table 5.2 used to evaluate the constants in equation 5.80.

Table 5.2 Constants for heat transfer from noncircular cylinders for use with Equation (5.80).

Geometry	Re_{df}	C	n
	$5\times10^3-10^5$	0.246	0.588
	$5\times10^3-10^5$	0.102	0.675
	$5\times10^3-1.95\times10^4$	0.160	0.638
	$1.95\times10^4-10^5$	0.0385	0.782
	$5\times10^3-10^5$	0.153	0.638
	$4\times10^3-1.5\times10^4$	0.228	0.731

Fand has shown that the heat-transfer coefficients from liquids to cylinders in cross flow may be better represented by the relation

$$Nu_{Df} = (0.35 + 0.56Re_{Df}^{0.52})Pr_f^{0.3} \quad 10^{-1} > Re_{Df} < 10^5 \quad \text{flow of liquid} \qquad 5.81$$

For equation 5.81 the properties at T_f

Still a more comprehensive relation is given by Churchill and Bernstein that is applicable over the complete range of available data:

For the flow of air, water, and liquid sodium

$$Nu_D = 0.3 + \frac{0.62Re_D^{\frac{1}{2}}Pr^{\frac{1}{3}}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{Re_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} \qquad 5.82$$

For $10^2 < Re_D < 10^7$, $Pe_D > 0.2$

Where:

$Pe_D = Re_D Pr$

$$Nu_D = \left[0.8237 - \ln\left(Pe_D^{\frac{1}{2}}\right)\right]^{-1} \qquad Pe_D < 0.2 \qquad 5.83$$

5.9.2 Spheres

McAdams recommends the following relation for heat transfer from spheres to a flowing gas:

- Flow of gas

$$\frac{hD}{k} = Nu_D = 0.37\left(\frac{u_\infty D}{v}\right)^{0.6} \qquad 17 < Re_D < 7 * 10^4 \qquad 5.84$$

All properties of fluid at film temperature T_f.

Achenbach has obtained relations applicable over a still wider range of Reynolds numbers for air with Pr =0.71:

$$Nu_D = 2 + (0.25Re_D + 3 * 10^{-4} Re_D^{1.6})^{\frac{1}{2}} \qquad \begin{cases} 100 < Re_D < 3 * 10^5 \\ Pr = 0.71 \end{cases} \qquad 5.84$$

$$Nu_D = 430 + a\, Re_D + b\, Re_D^2 + c\, Re_D^3 \qquad \begin{cases} 3 * 10^5 < Re_D < 5 * 10^6 \\ Pr = 0.71 \end{cases} \qquad 5.85$$

$$a = 0.5 \times 10^{-3} \qquad b = 0.25 \times 10^{-9} \qquad c = -3.1 \times 10^{-17}$$

All properties of fluid at film temperature T_f.

- Flow of liquid

For flow of liquids past spheres, the data of Kramers may be used to obtain the Correlation

$$Nu_D Pr^{-0.3} = 0.97 + 0.68\left(\frac{u_\infty D}{v}\right)^{0.5} \qquad 1 < Re_D < 2000 \qquad 5.86$$

All properties at film temperature T_f.

heat transfer from spheres to oil and water over a more extended range of Reynolds numbers from 1 to 200,000:

$$Nu_D Pr^{-0.3}\left(\frac{\mu_w}{\mu_\infty}\right)^{0.25} = 1.2 + 0.53 Re_D^{0.54} \qquad 1 < Re_D < 2 * 10^5 \qquad 5.87$$

where all properties are evaluated at free-stream conditions T_∞, except μ_w, which is evaluated at the surface temperature of the sphere T_w.

- Liquid and Gases

All the above data have been brought together by Whitaker to develop a single equation for gases and liquids flowing past spheres:

$$Nu_D = 2 + \left(0.4Re_D^{\frac{1}{2}} + 0.06Re_D^{\frac{2}{3}}\right)Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{\frac{1}{4}} \quad \begin{cases} 3.5 < Re_D < 8 * 10^4 \\ 0.7 < Pr < 380 \end{cases} \qquad 5.88$$

Properties in Equation (5.88) are evaluated at the free-stream temperature T_∞.

Example 5.11: Airflow Across Isothermal Cylinder

Air at 1 atm and 35◦C flows across a 5.0-cm-diameter cylinder at a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150◦C. Calculate the heat loss per unit length of the cylinder.

Solution:

We first determine the Reynolds number and then find the applicable constants from Table 5.1 for use with Equation (5.80). The properties of air are evaluated at the film temperature:

$$T_f = \frac{T_W + T_\infty}{2} = \frac{150+35}{2} = 92.5 + 273 = 365.5 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{(1.0132*10^5)}{(287)(365.5)} = 0.966 \text{ kg/m}^3$$

$Pr = 0.695$, $\mu = 2.14 * 10^{-5}$ kg/m.s, $k = 0.0312$ W/m.°C.

$$Re_D = \frac{\rho u_\infty D}{\mu} = \frac{(0.966)(50)(0.05)}{2.14*10^{-5}} = 1.129 * 10^5$$

From table 5.1 $C = 0.0266$, $n = 0.805$

$$Nu_{Df} = \frac{hD}{k_f} = C\left(\frac{u_\infty D}{v_f}\right)^n Pr_f^{\frac{1}{3}}$$

$$\frac{hD}{k_f} = (0.0266)(1.129 * 10^5)^{0.805}(0.695)^{\frac{1}{3}} = 275.1$$

$$h = \frac{(275.1)(0.0312)}{0.05} = 171.7 \text{ W/m}^2\text{.°C.}$$

$$\frac{q}{L} = h\pi D(T_w - T_\infty)$$

$$\frac{q}{L} = (171.7)(\pi)(0.05)(150 - 35) = 3100 \text{ W/m}$$

H.W

A fine wire having a diameter of $3.94*10^{-5}$ m is placed in a 1-atm airstream at 25◦C having a flow velocity of 50 m/s perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to 50◦C. Calculate the heat loss per unit length.

1. Use equation 5.80
2. Use equation 5.82

Example 5.12: Heat Transfer from Sphere

Air at 1 atm and 27◦C blows across a 12-mm-diameter sphere at a free-stream velocity of 4 m/s. A small heater inside the sphere maintains the surface temperature at 77◦C. Calculate the heat lost by the sphere.

Solution

Consulting Equation (5.88) we find that the Reynolds number is evaluated at the free-stream temperature. We therefore need the following

properties: at T_∞ =27◦C=300 K,

$Pr = 0.708$, $\mu_\infty = 2.14 * 10^{-5}$ kg/m.s, $k = 0.02624$ W/m.°C,

$v = 15.96 * 10^{-6}$ m²/s

At $T_w = 77$°C $= 350$ K, $\mu_w = 2.075 * 10^{-5}$ kg/m.s

$$Re_D = \frac{u_\infty D}{v} = \frac{(4)(0.012)}{15.96*10^{-6}} = 3059$$

$$Nu_D = 2 + \left(0.4Re_D^{\frac{1}{2}} + 0.06Re_D^{\frac{2}{3}}\right)Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{\frac{1}{4}}$$

$$Nu_D = 2 + \left((0.4)(3059)^{\frac{1}{2}} + (0.06)(3059)^{\frac{2}{3}}\right)(0.708)^{0.4}\left(\frac{1.8462}{2.075}\right)^{\frac{1}{4}} = 31.4$$

$$h = \frac{(Nu_D)(k)}{D} = \frac{(31.4)(0.02624)}{0.012} = 68.66 \text{ W/m}^2.°\text{C.}$$

$$q = h4\pi D^2(T_w - T_\infty)$$

$$q = (68.66)(4\pi)(0.006)^2(77 - 27) = 1.553 \text{ W}$$

5.10 Flow Across Tube Banks

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.